(The bar denotes normalized functions.) The energy matrix to be diagonalized will then have the form

$$\begin{bmatrix} \left\langle {}^{\frac{1}{2}}_{-\frac{1}{2}}\bar{J} \middle| H \middle| {}^{\frac{1}{2}}_{-\frac{1}{2}}\bar{J} \right\rangle & (1-\alpha_J{}^2)^{-1/2}\left\{ \left\langle {}^{\frac{1}{2}}_{-\frac{1}{2}}\bar{J} \middle| H \middle| {}^{\frac{1}{2}}_{\frac{1}{2}}\bar{J} \right\rangle - \alpha_J \left\langle {}^{\frac{1}{2}}_{-\frac{1}{2}}\bar{J} \middle| H \middle| {}^{\frac{1}{2}}_{-\frac{1}{2}}\bar{J} \right\rangle \right\} \\ (1-\alpha_J{}^2)^{1/2}\left\{ \left\langle {}^{\frac{1}{2}}_{-\frac{1}{2}}\bar{J} \middle| H \middle| {}^{\frac{1}{2}}_{\frac{1}{2}}\bar{J} \right\rangle - \alpha_J \left\langle {}^{\frac{1}{2}}_{-\frac{1}{2}}\bar{J} \middle| H \middle| {}^{\frac{1}{2}}_{-\frac{1}{2}}\bar{J} \right\rangle \right\} & (1-\alpha_J{}^2)^{-1}\left\{ \left\langle {}^{\frac{1}{2}}_{\frac{1}{2}}\bar{J} \middle| H \middle| {}^{\frac{1}{2}}_{\frac{1}{2}}\bar{J} \right\rangle - 2\alpha_J \left\langle {}^{\frac{1}{2}}_{-\frac{1}{2}}\bar{J} \middle| H \middle| {}^{\frac{1}{2}}_{\frac{1}{2}}\bar{J} \right\rangle \right. \\ & \left. + \alpha_J{}^2 \left\langle {}^{\frac{1}{2}}_{-\frac{1}{2}}\bar{J} \middle| H \middle| {}^{\frac{1}{2}}_{-\frac{1}{2}}\bar{J} \right\rangle \right\}. \end{bmatrix}$$

The various matrix elements appearing in this matrix can be expressed in terms of the parameters (7), (8) by means of Eqs. (A17), (A19).

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Measurement of the Triple Scattering Parameter R' in Proton-Proton Scattering at $137\frac{1}{2}$ MeV*

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The proton-proton triple scattering parameter R' has been measured at a laboratory energy of $137\frac{1}{2}$ MeV over a range of scattering angles θ_2. The following values were obtained: θ_2(laboratory)$=20°50'$, 0.562 ± 0.052; $25°26'$, 0.472 ± 0.054; $30°8'$, 0.375 ± 0.068; $35°16'$, 0.238 ± 0.084; $39°55'$, 0.251 ± 0.121. The stated errors include a 5% error in R' which is systematic from angle to angle. This has been combined quadratically with the other errors.

INTRODUCTION

This experiment continues the program of measuring the Wolfenstein triple-scattering parameters[1] in p-p scattering near 140 MeV. The depolarization parameter[2] D, rotation parameter[3] R, and the A parameter[4] have previously been measured. This article describes a measurement of the Wolfenstein parameter R' for p-p scattering at $137\frac{1}{2}$ MeV over the range of laboratory scattering angles 20 to 40°. The parameter R' relates the initial polarization in the plane of the second scattering and perpendicular to the incident direction of the component of polarization after scattering which is along the direction of the outgoing motion.

The experimental arrangement for the p-p measurement is shown in Fig. 1. A proton beam having its polarization vertical passes through a solenoid magnet. The polarization precesses 90° about the direction of motion, so that on leaving the solenoid the beam has a polarization in the horizontal plane and perpendicular to the direction of motion. (The sign of the incident polarization P_1 can be reversed by reversing the solenoid current.) The beam leaving the solenoid strikes the hydrogen target. Particles scattered through an angle θ_2 in the horizontal plane pass through a sector magnet which rotates the polarization through an angle near 90°, thereby changing a longitudinal component into a transverse component. This beam, defined by counters A, M, B, then strikes the analyzing scatterer. Particles scattered through an angle θ_3 in the vertical plane are detected by the counter telescopes CD and EF. The angles θ_3 of these telescopes can be reversed in sign (up or down).

The asymmetry e_{3s} is measured for the two senses of telescope counter position and for the two signs of incident polarization. R' is then related to the measured asymmetry by

$$e_{3s} = P_1 P_3 (R\cos\chi + R'\sin\chi)\,, \qquad (1)$$

where P_3 is the analyzing power and χ is the angle of spin rotation. (e_{3s} is defined as in Refs. 3 and 4.)

The apparatus and techniques used in this experiment are, with a few modifications, identical to those used for measuring R and A, and greater detail on various points may be found in Refs. 3 and 4.

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* Supported by the joint program of the U. S. Office of Naval Research and the U. S. Atomic Energy Commission.

[1] L. Wolfenstein, Ann. Rev. Nucl. Sci. **6**, 43 (1956).

[2] C. F. Hwang, T. R. Ophel, E. H. Thorndike, and Richard Wilson, Phys. Rev. **119**, 352 (1960).

[3] E. H. Thorndike, J. Lefrançois, and Richard Wilson, Phys. Rev. **120**, 1819 (1960).

[4] Stanley Hee and E. H. Thorndike, Phys. Rev. (to be published).

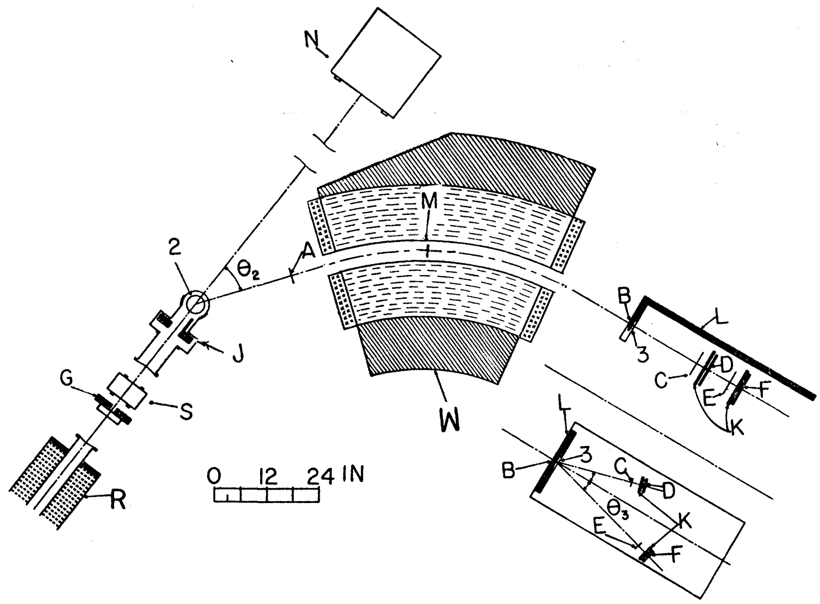

FIG. 1. Drawing of the experimental arrangement for R' showing: (2) hydrogen target, (3) analyzing scatterer, (A–F, M) scintillation counters, (G) main defining slits, (J) antiscattering slits, (K) copper absorbers, (L) iron shielding, (N) Faraday cup, (R) solenoid magnet, (S) ion chamber, (W) sector magnet. The inset (lower right) shows an elevation of the analyzing scattering.

PROCEDURES

The Beam

The beam is defined by slits $1\frac{1}{2}$ in. wide by 2 in. high located after the solenoid. The mean energy was $141\frac{1}{2}$ MeV and the polarization is estimated at 59%. Throughout the experiment the solenoid field was kept within 1% of 0.801×10^6 A turns, which value would rotate the polarization of a 142.7-MeV beam through 90°, and a $141\frac{1}{2}$ beam through 90.4°.

The solenoid produces a shift in the zero position of θ_3 on reversal of the current. Dipole windings were used as in the A experiment to reduce this effect. The rotation of the beam image by the solenoid rotates the horizontal energy dispersion of the Harvard synchrocyclotron into a vertical energy dispersion. The solenoid-dependent vertical energy dispersion was measured to be $1\pm\frac{1}{2}$ MeV. The energy is highest at the bottom when the solenoid is normal, and highest at the top when the solenoid is reversed. This dispersion necessitates a correction to the measurements.

Alignment and Background

The critical $\theta_3=0°$ alignment was performed using the slope method as described in Ref. 4. The misalignments were kept small by the use of dipole windings which bent the beam as a whole up or down. The solenoid-dependent beam shift was thus partially cancelled.

Background from other than hydrogen scattering was measured by evacuating the hydrogen target and increasing the copper absorbers in CD and EF to compensate for the change in energy. The backgrounds for the two telescopes were less than 2.6% at the smallest angle and less than 1.7% at the other angles. Random backgrounds were found to be negligible.

Scattering Angle and Spin Rotation Angle

The scattering angle and spin rotation angle were determined from profiles swept in the twice scattered beam. A "line counter" $\frac{1}{8}$ in. square was swept in front of the A counter and before and after the B counter. The first moments of these profiles were calculated and from these data the scattering angle and angle of bending in the sector magnet were deduced. The angle χ through which the polarization is rotated is related to the angle of bending by[5]

$$\chi=[(\mu_p-1)/(1-\beta^2)^{\frac{1}{2}}]\Omega, \tag{2}$$

where $\mu_p=2.793$ is the proton magnetic moment in nuclear magnetons, β is the ratio of the proton velocity to the velocity of light, and Ω is the angle of bending in the sector magnet. The results of the angle determinations are given in Table I.

P_1P_3

P_1P_3 is the product of the incident polarization and the analyzing power. It was inferred by indirect methods described in detail in Ref. 4 from previously measured values quoted in Ref. 3.

An interpolation to the analyzing energies of this

TABLE I. Scattering angle θ_2 and angle of spin rotation χ.

θ_2	χ
20° 50′±30′	91.7°±1.1°
25° 26′	90.7°
30°8′	92.8°
35° 16′	87.6°
39° 55′	88.0°

[5] V. Bargmann, L. Michel, and V. L. Telegdi, Phys. Rev. Letters 2, 435 (1959).

TABLE II. Corrections to R' from vertical energy variation at defining slits, and from an admixture of R.

θ_2	$\delta R'$ (Slit energy)	$\delta R'$ (Admixture of R)
20° 50′	-0.007 ± 0.003	-0.007 ± 0.005
25° 26′	-0.006 ± 0.003	-0.003 ± 0.005
30° 8′	-0.008 ± 0.004	-0.007 ± 0.003
35° 16′	-0.010 ± 0.005	$+0.006\pm0.004$
39° 55′	-0.008 ± 0.004	$+0.002\pm0.003$

experiment was performed with the previously measured P_1P_3 results of Ref. 3. These interpolated values are slightly in error because the angular divergence of the beam striking the analyzing scatterer changes slightly from the R and A experiments to the R' experiment. The defining counter B is located further from the hydrogen target in the R' experiment and this results in a narrower beam profile. The beam profiles taken in the R' alignment checks were compared to the θ_3 alignment profiles from the A experiment, and the error in P_1P_3 resulting from neglecting this effect was calculated. The error was found to be not greater than 2% at 20° and it decreased with scattering angle to 1% at 25°, 0.7% at 30°, 0.6% at 35°, ½% at 40°. Errors of these magnitudes were therefore added to other errors in P_1P_3. A further effect results from a possible difference in energy spread between the R and R' experiments. Range curves taken in the two experiments were compared and corrections to P_1P_3 (never more than 0.002) were made.

A possible difference in P_1 between experiments was treated as in Ref. 4. The ratio P_1 (R' experiment) divided by P_1 (R experiment) was found to be 0.99 ± 0.05. The quoted error includes, in addition to counting statistics of ±0.03, an allowance for systematic errors in the technique, as in Ref. 4.

The values of P_1P_3 used in the analysis were the interpolated values corrected for the difference in the shapes of the range curves, and decreased by the ratio of the P_1's. The error attributed to P_1P_3 was a quadratic combination of the random errors from counting statistics and the uncertainties of the interpolation with the 5% error (systematic from angle to angle) from P_1 and the error mentioned above from neglecting the angular divergence.

ANALYSIS AND RESULTS

Scattering Energy and Angle

The mean energy at the center of the full hydrogen target was determined to be $137\frac{1}{2}\pm1$ MeV from range curves taken in copper at each θ_2. The measurements here are based on the same range-energy relations as in previous experiments[3,4] and are directly comparable to energy measurements of previous triple scattering experiments performed at this laboratory. The stated error indicates the deviation of the various measurements from the mean, and does not include the uncertainty of the range-energy relations used. The energy varied with solenoid such that normal averaged higher than reverse by 0.9 MeV. The maximum variation was 1.2 MeV.

The θ_2 scattering angle was determined to $\pm30'$ from the profiles swept at the A counter. These profiles had a full width at half maximum of $\pm1.5°$.

Corrections and Errors

The measured asymmetries were corrected for backgrounds and for the θ_3 misalignments. The alignment correction did not exceed 0.006 in asymmetry for the two smaller θ_2 angles, and did not exceed 0.003 in asymmetry for the three larger angles. The corrected e_{3s} values for the two telescopes were averaged by weighting by the square of the reciprocal of the combined statistical and alignment errors.

R' was calculated using Eq. (1). The correction for the admixture of R was performed using the χ values from Table I and the R values from Ref. 3. The R values were interpolated to the θ_2 scattering angles used in this experiment. The corrections are small and differences due to the small energy difference between R and R' experiments are negligible. The corrections are listed in Table II.

The previously mentioned vertical energy variation at the defining slits necessitated a small correction to the R' values. These corrections are also listed in Table II.

The final quoted error[6] on R' is a quadratic combination of the random errors on e_{3s}, the errors on P_1P_3 discussed above, and the errors attributed to the corrections in Table II. The final error on R' thus includes quadratically a 5% error due to P_1 which is systematic from angle to angle.

The average beam energy, the second scattering angle, and the monitoring efficiency change on solenoid reversal. The errors resulting from this normal-reverse difference cancel on measuring the up-down asymmetry.

Any mechanical up-down asymmetry cancels on averaging over solenoid directions. It seemed desirable to show that any up-down asymmetries were small even though they cancelled. At 25°, the up-down asymmetry was measured for hydrogen scattering with solenoid off. The value obtained was -0.011 ± 0.015.

Mechanical misalignments other than θ_3 misalignments are negligible. θ_3 misalignments were treated as discussed previously. Undesired components of polarization, such as caused by the (slightly) incorrect solenoid current settings, were negligible in comparison to the quoted errors in R'. R is mixed in to first order since χ is not exactly 90°. This correction was discussed above.

Results

The consistency of the measurement was checked by comparing the e_{3s} values from the two telescopes.

[6] Errors quoted in this paper are intended as standard deviations.

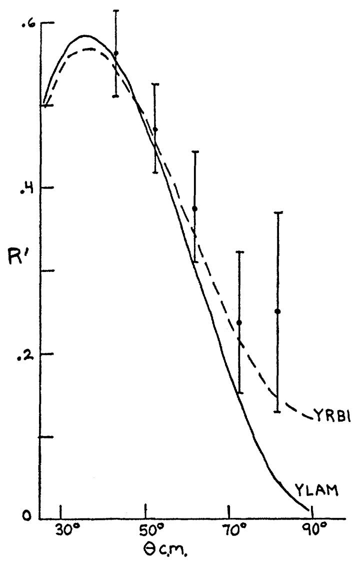

FIG. 2. R' ($137\frac{1}{2}$ MeV) for proton-proton scattering versus center-of-mass scattering angle. Shown also are two predictions of the Yale group[7] at 140 MeV.

At four of the five scattering angles the measurements differed by one standard deviation or less. All of the measurements differed by less than two standard deviations.

The final results of R' are listed in Table III and plotted in Fig. 2. The results are in agreement with phase shift analyses from preexisting data. Shown in Fig. 2 are two predictions of the Yale group[7] calculated at 140 MeV. YLAM represents the best fit to the data between 9.5 MeV and 345 MeV. The data of this experiment seem to favor YRB1, however. Presumably this indicates that minor adjustments must still be made to the fit.

TABLE III. R' values at $137\frac{1}{2}$ MeV.

θ_2 (lab)	R'
20° 50′	0.562 ± 0.052
25° 26′	0.472 ± 0.054
30° 8′	0.376 ± 0.068
35° 16′	0.238 ± 0.084
39° 55′	0.251 ± 0.121

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Recoil Energy Spectrum of the Sodium Ions Following the β^- Decay of Ne^{23}

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Following the beta decay of Ne^{23}, the sodium ions have a recoil energy distribution which is dependent on the nature of the decay. A precise measurement has been made of the recoil spectrum from 100 to 500 eV. From these data the electron-neutrino angular correlation coefficient α has been determined to be -0.33 ± 0.03, in good agreement with V-A interaction. The quoted error includes an uncertainty of only 2% due solely to the recoil measurements. The principal uncertainty in α arises from the errors quoted by Penning and Schmidt for their measurements of the β^--ray intensities to the ground and first excited states. If it is assumed that V-A interaction is valid and hence $\alpha=-1/3$, the β^--ray intensities to the ground and first excited states of Na^{23} may be calculated from the recoil data. Such calculations were made and the results are, respectively, $(67\pm1)\%$ and $(32\pm1)\%$. From the end point of the recoil spectrum, the decay energy to the ground state has been determined to be 4.383 ± 0.008 MeV.

I. INTRODUCTION

A NUMBER of measurements have been made on the recoil energy spectra of ions resulting from the beta decay of rare gases.[1,2] Data have also been obtained, making use of both β and recoil particles in coincidence.[3,4] The purpose of both sets of measurements was to elicit information on the angular correlation between the beta particle and the neutrino. Some uncertainty was present in the initial experiments, but later results established that the (V,A) interaction is dominant in beta decay.

The recent work of Johnson, Pleasonton, and Carlson[2] on He^6 has greatly increased the precision of recoil spectrometry. Advantage was taken of the knowledge

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